Section 1.5

Grade: «grade»

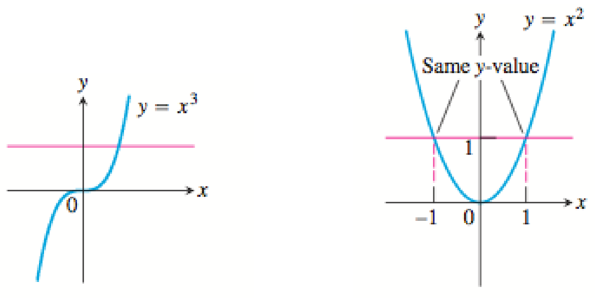
Subject: «subject»

Date: «date»

DEFINITION One-to-One Function

A function $f(x)$ is **one-to-one** on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.

The graph of a one-to-one function $y = f(x)$ can intersect any horizontal line at most once (the *horizontal line test*). If it intersects such a line more than once it assumes the same y-value more than once, and is therefore not one-to-one (Figure 1.33).



One-to-one: Graph meets each horizontal line once.

Not one-to-one: Graph meets some horizontal lines more than once.

Figure 1.33 Using the horizontal line test, we see that $y = x^3$ is one-to-one and $y = x^2$ is not.

1 Answer

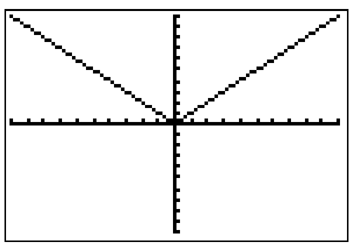
Yes

No

EXAMPLE 1 Using the Horizontal Line Test

Determine whether the functions are one-to-one.

$$(\mathbf{a})\ f(x) = |x|$$



Graph

no

2 Answer?

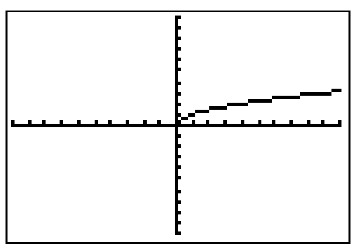
Yes

No

EXAMPLE 1 Using the Horizontal Line Test

Determine whether the functions are one-to-one.

(b) $g(x) = \sqrt{x}$



Graph

yes

Inverses

Table 1.13 Rental Charge versus Time

Time x (hours)	Charge y (dollars)
1	5.00
2	7.50
3	10.00
4	12.50
5	15.00
6	17.50

Table 1.14 Time versus Rental Charge

Charge x (dollars)	Time y (hours)
5.00	1
7.50	2
10.00	3
12.50	4
15.00	5
17.50	6

As Tables 1.13 and 1.14 suggest, composing a function with its inverse in either order sends each output back to the input from which it came. In other words, the result of composing a function and its inverse in either order is the **identity function,** the function that assigns each number to itself. This gives a way to test whether two functions f and g are inverses of one another. Compute $f \circ g$ and $g \circ f$. If $(f \circ g)(x) = (g \circ f)(x) = x$, then f and g are inverses of one another; otherwise they are not. The functions $f(x) = x^3$ and $g(x) = x^{1/3}$ are inverses of one another because $(x^3)^{1/3} = x$ and $(x^{1/3})^3 = x$ for every number x.

EXPLORATION 1 **Testing for Inverses Graphically**

For each of the function pairs below,

(a) Graph f and g together in a square window.

(b) Graph $f \circ g$. **(c)** Graph $g \circ f$.

What can you conclude from the graphs?

1. $f(x) = x^3, \quad g(x) = x^{1/3}$ yes

2. $f(x) = x, \quad g(x) = 1/x$ no

3. $f(x) = 3x, \quad g(x) = x/3$ yes

4. $f(x) = e^x, \quad g(x) = \ln x$ yes

Writing f^{-1} as a Function of x

1. Solve the equation $y = f(x)$ for x in terms of y.
2. Interchange x and y. The resulting formula will be $y = f^{-1}(x)$.

EXAMPLE 2 Finding the Inverse Function

Show that the function $y = f(x) = -2x + 4$ is one-to-one and find its inverse function.

$y = -2x + 4$

$x = -2y + 4$

$x - 4 = -2y$

$y = -\frac{1}{2}x + 2$

DEFINITION Base *a* Logarithm Function

The **base *a* logarithm function** $y = \log_a x$ is the inverse of the base a exponential function $y = a^x$ $(a > 0, a \neq 1)$.

Inverse Properties for a^x and $\log_a x$

1. Base a: $a^{\log_a x} = x$, $\log_a a^x = x$, $a > 1, x > 0$
2. Base e: $e^{\ln x} = x$, $\ln e^x = x$, $x > 0$

Solve for x: (a) $\ln x = 3t + 5$

power

equal to

$\log_e x = 3t + 5$

base

$e^{3t+5} = x$

3 Answer (nearest hundredth)?

(b) $e^{2x} = 10$

$\ln 10 = 2x$

$\frac{1}{2}\ln 10 = x$

Properties of Logarithms

For any real numbers $x > 0$ and $y > 0$,

1. *Product Rule*: $\log_a xy = \log_a x + \log_a y$
2. *Quotient Rule*: $\log_a \frac{x}{y} = \log_a x - \log_a y$
3. *Power Rule*: $\log_a x^y = y \log_a x$

EXPLORATION 2 **Supporting the Product Rule**

Let $y_1 = \ln(ax)$, $y_2 = \ln x$, and $y_3 = y_1 - y_2$.

1. Graph y_1 and y_2 for $a = 2, 3, 4$, and 5. How do the graphs of y_1 and y_2 appear to be related?
2. Support your finding by graphing y_3.
3. Confirm your finding algebraically.

$a=2$

$y = \ln 2x - \ln x$

$= \ln \frac{2x}{x}$

$= \ln 2$

$y \approx .7$

Change of Base Formula

$$\log_a x = \frac{\ln x}{\ln a} \qquad \frac{\log_{10} x}{\log_{10} a}$$

EXAMPLE 5 Graphing a Base *a* Logarithm Function

Graph $f(x) = \log_2 x$.

$$= \frac{\ln x}{\ln 2}$$

4 Answer (nearest tenth in years)?

EXAMPLE 6 Finding Time

Sarah invests $1000 in an account that earns 5.25% interest compounded annually. How long will it take the account to reach $2500?

how many times a year

$$\text{amount} = \text{initial}(1+r)^{t(1)}$$

$$2500 = 1000(1.0525)^t$$

$$1.0525^t = 2.5$$

$$\log_{1.0525} 2.5 = t$$

$$\frac{\ln 2.5}{\ln 1.0525} = t$$

5 Answer (nearest hundredth in trillion cubic feet)?

EXAMPLE 7 Estimating Natural Gas Production

Table 1.15 shows the annual number of cubic feet in trillions of natural gas produced by Saudi Arabia for several years.

Find the natural logarithm regression equation for the data in Table 1.15 and use it to estimate the number of cubic feet of natural gas produced by Saudi Arabia in 2002. Compare with the actual amount of 2.00 trillion cubic feet in 2002.

Table 1.15 Saudi Arabia's Natural Gas Production

Year	Cubic Feet (trillions)
1997	1.60
1998	1.65
1999	1.63
2000	1.76
2001	1.90

Source: Statistical Abstract of the United States, 2004–2005.

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Radian Measure

The **radian measure** of the angle ACB at the center of the unit circle (Figure 1.40) equals the length of the arc that ACB cuts from the unit circle.

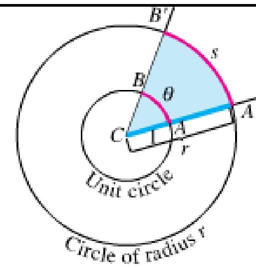


Figure 1.40 The radian measure of angle ACB is the length θ of arc AB on the unit circle centered at C. The value of θ can be found from any other circle, however, as the ratio s/r.

$s = \theta r$

EXAMPLE 1 Finding Arc Length

Find the length of an arc subtended on a circle of radius 3 by a central angle of measure $2\pi/3$.

1 Answer (nearest hundredth)?

$$s = \frac{2\pi}{\cancel{3}}(\cancel{3}) = 2\pi$$

$$\approx 6.28$$

sine: $\sin\theta = \frac{y}{r}$

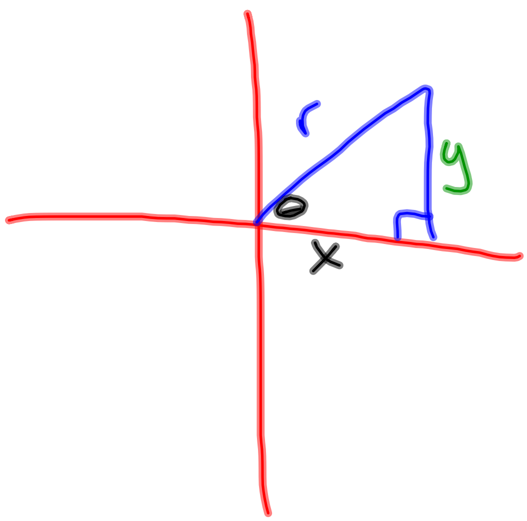
cosine: $\cos\theta = \frac{x}{r}$

tangent: $\tan\theta = \frac{y}{x}$

cosecant: $\csc\theta = \frac{r}{y}$

secant: $\sec\theta = \frac{r}{x}$

cotangent: $\cot\theta = \frac{x}{y}$



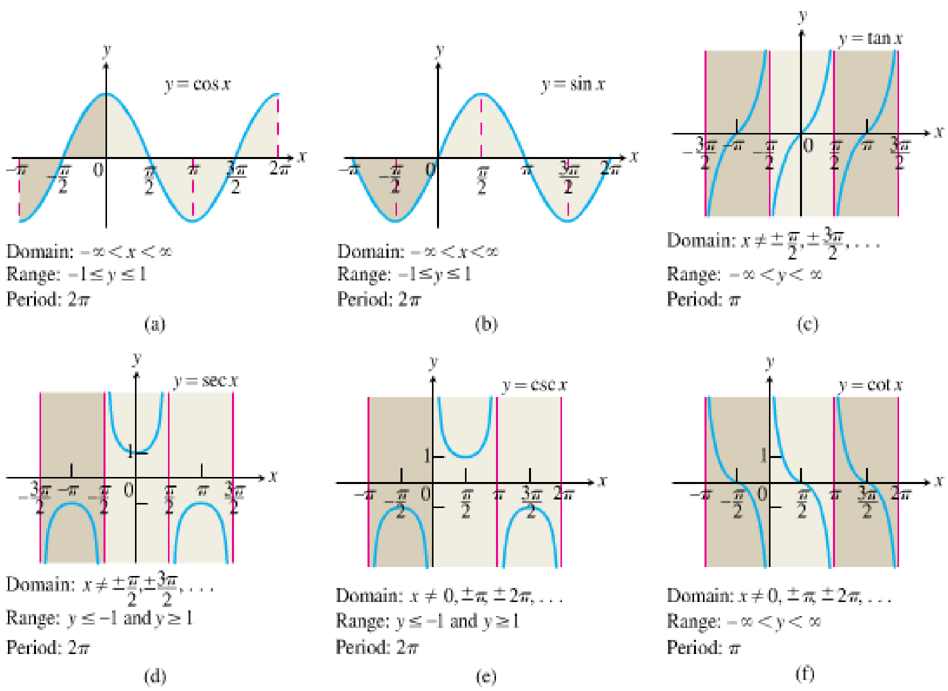


Figure 1.42 Graphs of the (a) cosine, (b) sine, (c) tangent, (d) secant, (e) cosecant, and (f) cotangent functions using radian measure.

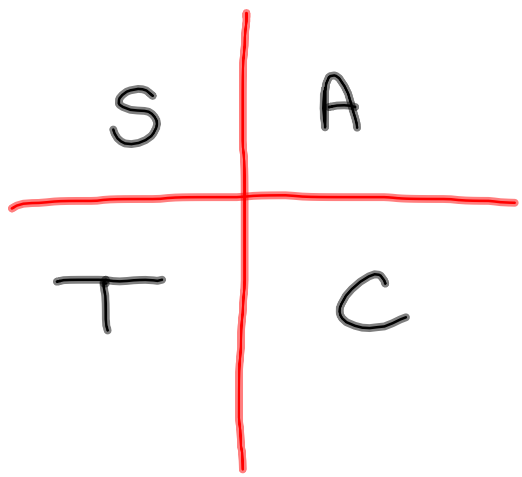
Graph these functions on your grapher and compare with the sketches. Use Zoom 7 for your window.

EXAMPLE 3 Finding Trigonometric Values

Find all the trigonometric values of θ if $\sin\theta = -3/5$ and $\tan\theta < 0$.

2 Cosine?

$\cos\theta = \frac{x}{r} = \frac{4}{5}$



3 Tangent?

$$\tan\theta = \frac{y}{x} = \frac{-3}{4}$$

4 Cosecant?

$$\csc\theta = \frac{r}{y} = -\frac{5}{3}$$

5 Secant?

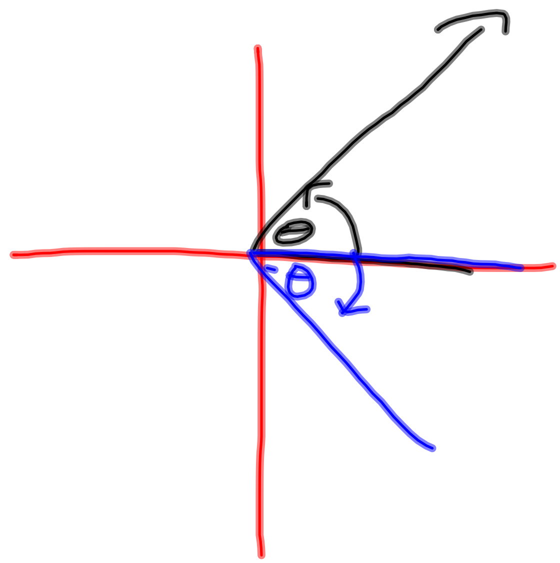
$$\sec\theta = \frac{r}{x} = \frac{5}{4}$$

6 Cotangent?

$$\cot\theta = \frac{x}{y} = \frac{4}{-3}$$

$\sin(-\theta) = -\sin\theta$ odd

$\cos(-\theta) = \cos\theta$ even



Transformations of Trigonometric Graphs

The rules for shifting, stretching, shrinking, and reflecting the graph of a function apply to the trigonometric functions. The following diagram will remind you of the controlling parameters.

$$y = af(b(x + c)) + d$$

- a: Vertical stretch or shrink; reflection about x-axis
- d: Vertical shift
- b: Horizontal stretch or shrink; reflection about y-axis
- c: Horizontal shift

The general sine function or **sinusoid** can be written in the form

$$f(x) = A \sin\left[\frac{2\pi}{B}(x - C)\right] + D,$$

where $|A|$ is the *amplitude*, $|B|$ is the *period*, C is the *horizontal shift*, and D is the *vertical shift*.

EXAMPLE 4 Graphing a Trigonometric Function

Determine the **(a)** period, **(b)** domain, **(c)** range, and **(d)** draw the graph of the function $y = 3\cos(2x - \pi) + 1$.

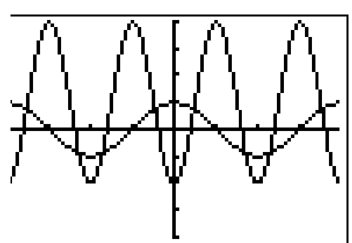
$y = 3\cos\left[2\left(x - \frac{\pi}{2}\right)\right] + 1$

Use GSP to understand this function.

a) $\frac{2\pi}{2} = \pi$

b) $(-\infty, \infty)$

c) $[-2, 4]$



Graph

DEFINITIONS **Inverse Trigonometric Functions**

Function	Domain	Range
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \sec^{-1} x$	$\lvert x\rvert \ge 1$	$0 \le y \le \pi, y \ne \frac{\pi}{2}$
$y = \csc^{-1} x$	$\lvert x\rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$
$y = \cot^{-1} x$	$-\infty < x < \infty$	$0 < y < \pi$

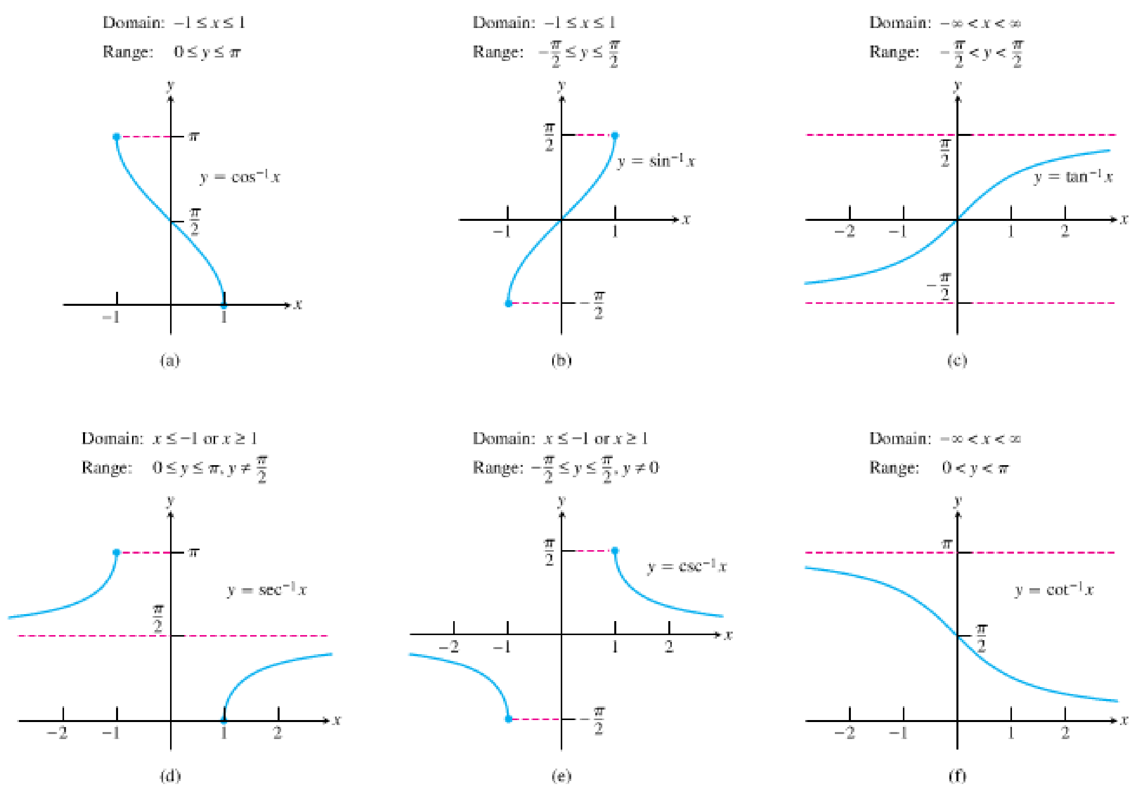


Figure 1.48 Graphs of (a) $y = \cos^{-1} x$, (b) $y = \sin^{-1} x$, (c) $y = \tan^{-1} x$, (d) $y = \sec^{-1} x$, (e) $y = \csc^{-1} x$, and (f) $y = \cot^{-1} x$.

7 Answer #1(nearest thousandth)?

EXAMPLE 8 Using the Inverse Trigonometric Functions

Solve for x.

(a) $\sin x = 0.7$ in $0 \leq x < 2\pi$

$\sin x = 0.7$

$x = \sin^{-1} 0.7$

$x \approx .775$

8 Answer #2(nearest thousandth)?

EXAMPLE 8 Using the Inverse Trigonometric Functions

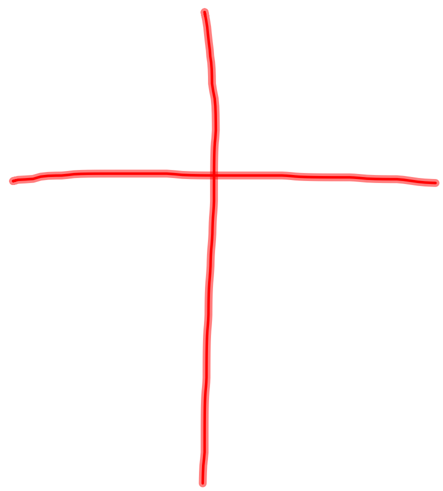
Solve for x.

(a) $\sin x = 0.7$ in $0 \le x < 2\pi$

$x = \pi - \sin^{-1} 0.7$

≈ 2.366

π



(b) $\tan x = -2$ in $-\infty < x < \infty$

$\tan^{-1} -2 \approx -1.107 + k\pi$